MA206X AY26-2 — Block II Practice Review

Statistical Inference (Lessons 17–25)

This review is for practice only and is not graded.

Work each problem completely. You are authorized your WPR2 SRC and R-lite interpreter (`qt`, `pt`, `qnorm`, `pnorm`). State hypotheses using population parameters, show all work for test statistics, report p-values using R-lite commands, and write conclusions in context.

Concept Check

Answer each of the following. These are quick checks of key definitions and properties.

1. True or False: The Central Limit Theorem requires the population to be normally distributed.

2. True or False: A 95% confidence interval means there is a 95% probability that the true parameter lies within the interval.

3. Fill in the blank: The standard error of $\bar{X}$ is _____.

4. True or False: Failing to reject H_0 proves that the null hypothesis is true.

5. Fill in the blank: A Type I error occurs when we _____.

6. True or False: If a 90% CI for μ is $(52, 68)$, we would reject $H_0 : \mu = 70$ at $\alpha = 0.10$.

7. Fill in the blank: For a one-proportion z-test, the success–failure condition requires _____.

8. True or False: In a two-proportion hypothesis test, we use the pooled proportion $\hat{p} = \frac{x_1+x_2}{n_1+n_2}$ in the standard error.

9. True or False: A paired t-test is appropriate when comparing two independent groups.

10. Fill in the blank: For a two-sample t-test (by hand), we use the conservative degrees of freedom $df =$ _____.

CLT & Sampling Distributions (Lesson 17)

Problem 1. A coffee shop's transaction times have a mean of $\mu = 4.5$ minutes and standard deviation $\sigma = 1.2$ minutes. A random sample of $n = 36$ transactions is selected.

(a) Describe the sampling distribution of $\bar{X}$.

(b) Find $P(\bar{X} > 4.8)$.

(c) Find $P(4.3 < \bar{X} < 4.7)$.

Problem 2. A survey finds that 60% of consumers in a large city prefer online shopping. A random sample of $n = 200$ consumers is selected.

(a) Verify the CLT conditions for $\hat{p}$ and describe the sampling distribution.

(b) Find $P(\hat{p} > 0.65)$.

(c) How large a sample would be needed so that the standard error of $\hat{p}$ is at most 0.02?

Confidence Intervals (Lessons 18–19)

Problem 3. A bakery wants to estimate the average weight of its sourdough loaves. A random sample of 16 loaves yields $\bar{x} = 510$ grams and $s = 12$ grams. Assume loaf weights are approximately normal.

(a) Construct a 95% confidence interval for the true mean loaf weight.

(b) Interpret the interval in context.

(c) If the bakery wants the margin of error to be at most 3 grams (at 95% confidence), how many loaves should they sample?

Problem 4. A university surveys 500 randomly selected students and finds that 340 use the campus library at least once per week.

(a) Construct a 90% confidence interval for the true proportion of students who use the library weekly.

(b) Interpret the interval in context.

(c) Without computing, would a 99% CI be wider or narrower? Explain.

One-Sample t-Test (Lesson 21)

Problem 5. Customers at a restaurant complain that wait times exceed the advertised average of 20 minutes. A random sample of 25 visits yields $\bar{x} = 22.4$ minutes and $s = 5.1$ minutes.

(a) State the hypotheses to test whether the average wait time exceeds 20 minutes.

(b) At $\alpha = 0.05$, conduct the hypothesis test. Show the test statistic, p-value, decision, and conclusion.

(c) Construct a 95% confidence interval for μ and confirm it is consistent with your test decision.

Problem 6. A laptop manufacturer advertises 8 hours of battery life. A consumer group tests 12 randomly selected laptops and finds $\bar{x} = 7.5$ hours with $s = 0.9$ hours. Test at $\alpha = 0.05$ whether the average battery life differs from the advertised claim.

(a) State the hypotheses.

(b) Conduct the test and state your conclusion.

(c) Construct a 95% CI for the true mean battery life and verify consistency with your test.

One-Proportion z-Test (Lesson 22)

Problem 7. An online retailer claims that at least 80% of orders are delivered on time. In a random sample of 400 orders, 300 arrived on time. Test the claim at $\alpha = 0.05$.

(a) State the hypotheses.

(b) Verify the conditions for the test.

(c) Conduct the test and state your conclusion.

(d) Construct a 95% confidence interval for the true on-time delivery rate.

Problem 8. A university claims that at least 85% of its students graduate within four years. A sample of 500 recent students reveals that 405 graduated on time. Test at $\alpha = 0.01$.

(a) State hypotheses, conduct the test, and state your conclusion.

(b) Is the result practically significant? The university's accreditation requires at least 80%.

Two-Sample t-Test (Lesson 23)

Problem 9. A university wants to compare weekly study hours between STEM and humanities majors. Independent random samples yield:

Group	n	$\bar{x}$	s
STEM	30	18.5 hrs	4.2 hrs
Humanities	35	15.8 hrs	3.6 hrs

(a) State the hypotheses to test whether average study hours differ between the two groups.

(b) Compute the test statistic, p-value, and state your conclusion at $\alpha = 0.05$.

(c) Construct a 95% CI for $\mu_1 - \mu_2$.

Problem 10. A transportation analyst compares average commute times (in minutes) between two cities:

City	n	$\bar{x}$	s
City A	40	32.5	8.3
City B	45	28.1	7.6

(a) Test at $\alpha = 0.05$ whether City A residents have a longer average commute than City B residents.

(b) Construct a 95% CI for the difference in mean commute times.

Paired t-Test (Lesson 24)

Problem 11. A company wants to evaluate whether ergonomic keyboards improve typing speed. Eight employees are tested before and after switching to the new keyboards. The differences (after − before) yield $\bar{d} = 5.6$ wpm, $s_d = 3.8$ wpm.

(a) Explain why a paired t-test is appropriate here rather than a two-sample t-test.

(b) Test at $\alpha = 0.05$ whether the new keyboards increased typing speed.

(c) Construct a 95% CI for the true mean improvement in typing speed.

Problem 12. A counseling center tests whether a new meditation program reduces student anxiety scores. Ten students are measured before and after the 6-week program. The differences (before − after) yield $\bar{d} = 3.2$ points, $s_d = 6.5$ points.

(a) Test at $\alpha = 0.05$ whether the program reduced anxiety scores.

(b) Construct a 95% CI for μ_d and verify consistency.

Two-Proportion z-Test (Lesson 25)

Problem 13. Two clinics compare patient satisfaction rates. Clinic A surveyed 250 patients and found 180 satisfied. Clinic B surveyed 200 patients and found 120 satisfied. Test at $\alpha = 0.05$ whether the satisfaction rates differ.

(a) State the hypotheses.

(b) Conduct the test.

(c) Construct a 95% CI for $p_1 - p_2$.

Problem 14. Two coffee chains survey customers about a new seasonal blend. Chain A: 156 out of 240 prefer the new blend. Chain B: 120 out of 200 prefer the new blend. Test at $\alpha = 0.05$ whether the preference rates differ.

(a) Conduct the full hypothesis test.

(b) Construct a 95% CI for $p_1 - p_2$ and verify consistency.

Mixed Practice

Problem 15. For each scenario, identify the most appropriate test and define the parameter of interest. You do **not** need to carry out the test.

(a) A hospital wants to know if the average patient stay differs from 4.5 days. They sample 30 patients.

(b) A polling firm wants to test whether more than 55% of voters support a ballot measure. They poll 800 voters.

(c) A fitness center measures resting heart rate of 20 members before and after a 12-week exercise program.

(d) An economist compares average household spending between two cities using independent samples of 50 households from each.

(e) A researcher compares the proportion of defective items from two different factories.

Problem 16. A food safety inspector tests $H_0 : \mu = 40°\text{F}$ (food is stored at proper temperature) vs. $H_a : \mu > 40°\text{F}$ (food is stored too warm) at $\alpha = 0.05$.

(a) Describe a Type I error in this context. What is its probability?

(b) Describe a Type II error in this context.

(c) Which error is more serious here? How could you reduce it?

Problem 17. A 95% confidence interval for μ is $(48.2,\ 55.8)$.

(a) Would you reject $H_0 : \mu = 46$ vs. $H_a : \mu \neq 46$ at $\alpha = 0.05$? Explain.

(b) Would you reject $H_0 : \mu = 50$ vs. $H_a : \mu \neq 50$ at $\alpha = 0.05$? Explain.

(c) Can you determine whether you would reject $H_0 : \mu = 46$ at $\alpha = 0.01$? Explain.

Problem 18. A grocery store claims the average checkout time is at most 7 minutes. A random sample of 20 transactions yields $\bar{x} = 8.2$ minutes and $s = 2.4$ minutes. Test at $\alpha = 0.05$.

(a) Conduct the full hypothesis test.

(b) Construct a 90% CI for the true average checkout time.

Problem 19. A tutoring company tests whether its program improves student exam scores. Fifteen students are tested before and after the program. Summary statistics for the differences (after − before): $\bar{d} = 4.8$ points, $s_d = 7.2$ points.

(a) Identify the appropriate test and justify your choice.

(b) Conduct the test at $\alpha = 0.05$.

(c) Construct a 95% CI for the true mean improvement.

Problem 20. A large university ($n = 5000$ students) tests whether the average GPA differs from 3.00 and finds $\bar{x} = 3.02$ with $s = 0.45$.

(a) Compute the test statistic and p-value for $H_0 : \mu = 3.00$ vs. $H_a : \mu \neq 3.00$.

(b) What is your decision at $\alpha = 0.05$?

(c) The approximate 95% CI is (3.008, 3.032). Is this result practically meaningful? Explain.

(d) What feature of this problem drives the small p-value despite the tiny effect?