MA206X Inference Summary

Hypothesis Tests and Confidence Intervals

Hypothesis Tests – Means and Paired

	1-Sample Mean (Large)	1-Sample Mean (Small)	2-Sample Mean (Large)	2-Sample Mean (Small)	Paired Mean
Parameter	μ	μ	$\mu_1 - \mu_2$	$\mu_1 - \mu_2$	μ_d
H_0	$\mu = \mu_0$	$\mu = \mu_0$	$\mu_1 - \mu_2 = \Delta_0$	$\mu_1 - \mu_2 = \Delta_0$	$\mu_d = \Delta_0$
H_a	$\mu \neq, <, > \mu_0$	$\mu \neq, <, > \mu_0$	$\mu_1 - \mu_2 \neq, <, > 0$	$\mu_1 - \mu_2 \neq, <, > 0$	$\mu_d \neq, <, > 0$
Test Statistic	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$	$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$	$t = \frac{\bar{d} - \Delta_0}{s_d/\sqrt{n}}$
Distribution	$N(0,1)$	t_{n-1}	$N(0,1)$	$t_{\min(n_1-1,\, n_2-1)}$	t_{n-1}
Left-tailed p	`pnorm(z)`	`pt(t,df)`	`pnorm(z)`	`pt(t,df)`	`pt(t,df)`
Right-tailed p	`1-pnorm(z)`	`1-pt(t,df)`	`1-pnorm(z)`	`1-pt(t,df)`	`1-pt(t,df)`
Two-tailed p	`2*(1-pnorm(|z|))`	`2*(1-pt(|t|,df))`	`2*(1-pnorm(|z|))`	`2*(1-pt(|t|,df))`	`2*(1-pt(|t|,df))`
Conditions	$n \geq 30$	Normal pop or $n \geq 30$	$n_1, n_2 \geq 30$	Both pops $\sim$ Normal	Diffs $\sim$ Normal or $n \geq 30$

Hypothesis Tests – Proportions and ANOVA

	One Proportion	Two Proportions	One-Way ANOVA
Parameter	p	$p_1 - p_2$	$\mu_1, \dots, \mu_k$
H_0	$p = p_0$	$p_1 - p_2 = 0$	$\mu_1 = \dots = \mu_k$
H_a	$p \neq, <, > p_0$	$p_1 - p_2 \neq, <, > 0$	at least one μ_i differs
Test Statistic	$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$	$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(1/n_1 + 1/n_2)}}, \quad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$	$F = \frac{MSTr}{MSE}$
Distribution	$N(0,1)$	$N(0,1)$	$F_{k-1,\, N-k}$
Left-tailed p	`pnorm(z)`	`pnorm(z)`	—
Right-tailed p	`1-pnorm(z)`	`1-pnorm(z)`	`1-pf(F,df1,df2)`
Two-tailed p	`2*(1-pnorm(|z|))`	`2*(1-pnorm(|z|))`	—
Conditions	$np_0 \geq 10$, $n(1-p_0) \geq 10$	$n_i\hat{p}_i \geq 10$, $n_i(1-\hat{p}_i) \geq 10$	Indep. samples; equal var.; normal within

Decision rule: $p \leq \alpha \Rightarrow$ Reject H_0. $p > \alpha \Rightarrow$ Fail to reject H_0.

Confidence Intervals – Means and Paired

	1-Sample Mean (Large)	**1-Sample Mean (Small)**	**Two-Sample Mean**	**Paired Mean**
Parameter	μ	μ	$\mu_1 - \mu_2$	μ_d
Formula	$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	$\bar{x} \pm t_{\alpha/2,n-1} \cdot \frac{s}{\sqrt{n}}$	$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$\bar{d} \pm t_{\alpha/2,n-1} \cdot \frac{s_d}{\sqrt{n}}$
df	—	$n-1$	$\min(n_1-1,\, n_2-1)$	$n-1$
Conditions	$n \geq 30$	Normal pop or $n \geq 30$	$n_1, n_2 \geq 30$ (or Normal pops)	Diffs $\sim$ Normal or $n \geq 30$

Confidence Intervals – Proportions

	One Proportion	**Two Proportions**
Parameter	p	$p_1 - p_2$
Formula	$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$
Conditions	$n\hat{p} \geq 10,\ n(1-\hat{p}) \geq 10$	$n_i\hat{p}_i \geq 10,\ n_i(1-\hat{p}_i) \geq 10$